

CP violation in gauge theories

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We define the CP transformation properties of scalars, fermions and vectors in a gauge theory and show that only three types of interactions can lead to CP violation: scalar interactions, fermion-scalar interactions and FF which is associated with the strong CP problem and which involves only the gauge fields. For technicolor theories this implies the absence of CP violation within perturbation theory.

a. Introduction The origin of CP violation is one of the current important questions in high-energy physics. There are three types of renormalizable interactions that are commonly considered and that violate CP: Yukawa couplings, scalar interactions and WW -type terms associated with global chiral anomalies. These are, in fact, the *only* CP-violating renormalizable interactions [1]. In this short note we consider some consequences of this fact relevant for the observability of *non*-Standard Model CP-violating effects; in addition we provide a simplified proof of the absence of (perturbative) CP-violating effects within the gauge sector of a general renormalizable theory with vectors and scalars.

Of course, the most popular extensions of the SM, viz., supersymmetric models, have scalar fields with Yukawa interactions and self-interactions so this result does not restrict them much. However, in view of this theorem, the *only* explicit CP-violating mechanism that occurs in field theories *without* scalar fields is due to global chiral anomalies. Any other mechanism will be due to spontaneous symmetry breaking associated with the formation of scalar condensates, as are presumed to form in technicolor models. Such theories necessarily involve strong gauge interactions and so are difficult to deal with, at least in a calculable manner. However, if a sufficient number of such scalars are present, they can in principle lead to CP-violating effects. [2] This contrasts with spontaneous parity violation, which cannot occur in theories with only vector-like gauge-interactions of fermions. [3] Thus, it is the spontaneous breaking of charge-conjugation that underlies this kind of CP-violation.

As an application of this result we investigate the observability of virtual CP-violating effects generated by physics beyond the standard model in the case where the heavy physics is described by a weakly-coupled, natural gauge theory. In the following the label “natural” will be used in the technical sense: the smallness of coefficients are protected by symmetries [5]. We argue that CP-violating effects are very hard to distinguish from those produced by the Standard Model so that, at least for this class of heavy theories, CP-violating processes are *not* a sensitive probe of physics *beyond* the Standard Model.¹

For clarity we will first present the proof that there is a natural definition of the CP transformation that reduces to the usual one and for which all renormalizable gauge interactions are invariant. The second part of the paper contains the phenomenological application of the result.

b. Properties of the group generators Although the definition of the CP transformations is not unique [4], the one provided below has the virtue of generalizing the one used in the Standard Model. The simplest expressions for the transformation of the fields are obtained when the group generators are expressed in a Cartan basis. We denote by H_i the Cartan generators and by E_α the generator corresponding to the root α , then

¹These arguments presuppose that there is no direct observation of the heavy excitations, that is, that the scale of new physics is not directly accessible. In cases where this is not true – as, for example, occurs in some supersymmetric extensions of the Standard Model – the observability of CP-violating effects must be determined on a case-by-case basis.

$$\begin{aligned} [H_i, H_j] &= 0 & [E_\alpha, E_{-\alpha}] &= \alpha_i H_i \\ [H_i, E_\alpha] &= \alpha_i E_\alpha & [E_\alpha, E_\beta] &= N_{\alpha, \beta} E_{\alpha+\beta} \quad (\alpha + \beta \text{ root}); \end{aligned} \quad (1)$$

we will not need the explicit expression for $N_{\alpha, \beta}$, only the property [6]

$$N_{\alpha, -\beta} = -N_{-\alpha, \beta}. \quad (2)$$

We will denote the simple roots by \mathbf{a}_r and the fundamental weights by μ_r , then $\mu_r \cdot \mathbf{a}_s = \delta_{rs}$.

We will show that for any irreducible representation (irrep) it is possible to choose a basis where all the generators are real and such that $E_{-\alpha}^T = E_\alpha$. To this end we note first that the Cartan generators can be taken to be Hermitian and diagonal, hence they are real. It then follows that the complex conjugate E_α^* corresponds to the same root as E_α and therefore [7] $E_\alpha^* = \lambda_\alpha E_\alpha$ for some complex numbers λ_α obeying

$$|\lambda_\alpha| = 1, \quad \lambda_\alpha \lambda_\beta = \lambda_{\alpha+\beta} \text{ when } \alpha + \beta \text{ is a root} \quad (3)$$

(the second result follows from the commutators); as a special case we have $\lambda_\alpha \lambda_{-\alpha} = 1$. Noting now that $E_\alpha^\dagger = E_{-\alpha}$ we obtain

$$E_{-\alpha} = \lambda_\alpha E_\alpha^T \quad (4)$$

where T denotes transposition.

Using the expansion of α in terms of the simple roots $\alpha = \sum_r c_r \mathbf{a}_r$ together with (3) implies

$$\lambda_\alpha = \prod_r \lambda_{\mathbf{a}_r}^{c_r} \quad (5)$$

Furthermore, since $|\lambda| = 1$ we can write $\lambda_{\mathbf{a}_r} = \exp(i\phi_r)$ which we use to define the unitary matrix

$$U = \exp \left(\frac{i}{2} \sum_{r,i} \phi_r H_i \cdot \mu_r^i \right) \quad (6)$$

where, as before, μ_r denote the fundamental weights.

We now use U to generate a new basis. We clearly have $H'_i = U H_i U^\dagger = H_i$ while the roots are rescaled appropriately,

$$E'_\alpha = U E_\alpha U^\dagger = \exp \left(+\frac{i}{2} \sum_r \phi_r c_r \right) E_\alpha = \prod_r \lambda_{\mathbf{a}_r}^{c_r/2} E_\alpha = \lambda_\alpha^{1/2} E_\alpha. \quad (7)$$

Then

$$E'^\dagger_\alpha = \left(\lambda_\alpha^{1/2} \right)^* \lambda_\alpha E_\alpha = \sqrt{\lambda_\alpha} E_\alpha = E'_\alpha \quad (8)$$

In this basis all the generators are real and, dropping the primes,

$$H_i = H_i^* = H_i^T \quad E_\alpha = E_\alpha^* = E_\alpha^T \quad (9)$$

proving our assertion [1].

c. CP transformation for the fields We now consider a gauge theory with gauge fields W_μ , fermions ψ assumed to be contained in a single multiplet that in general transforms reducibly, and scalars ϕ also lumped into one multiplet. We define the CP transformation of the fields as follows,

$$\begin{aligned} \psi &\rightarrow C\psi^* & W_\mu^{(i)} &\rightarrow -W^{(i)\mu} \\ \phi &\rightarrow \phi^* & W_\mu^{(\alpha)} &\rightarrow -W^{(-\alpha)\mu} \end{aligned} \quad (10)$$

where C denotes the charge conjugation matrix (acting on the Lorenz indices of ψ). It is understood that the arguments of the fields become their P transforms and that the fields are in a basis whose gauge generators obey (9). It is easy to see that for the standard model (10) reduces to the usual expressions, in particular the change in sign of the root α corresponds to a replacement of positive charged fields by negative charged ones.

With these definitions and using (9) the currents are seen to obey

$$\begin{aligned}
j_{L/R\mu}^{(i)} &= \bar{\psi} H_i \gamma_\mu P_{L/R} \psi \xrightarrow{CP} -j_{L/R}^{(i)\mu} \\
j_{L/R\mu}^{(\alpha)} &= \bar{\psi} E_\alpha \gamma_\mu P_{L/R} \psi \xrightarrow{CP} -j_{L/R}^{(-\alpha)\mu}
\end{aligned} \tag{11}$$

which implies that the fermionic kinetic Lagrangian $\bar{\psi} \not{D} \psi$ is invariant.

Denoting the curvature associated with W_μ by $W_{\mu\nu}$ and using (2) and (10), we get

$$W_{\mu\nu}^{(i)} \rightarrow -W^{(i)\mu\nu} \quad W_{\mu\nu}^{(\alpha)} \rightarrow -W^{(-\alpha)\mu\nu} \tag{12}$$

so that the gauge Lagrangian $-\frac{1}{4}W_{\mu\nu}^2$ is also invariant².

An explicit mass term for the fermions (if present) transforms according to

$$\bar{\psi} M \psi \xrightarrow{CP} \bar{\psi} M^T \psi. \tag{13}$$

where we have assumed that possible terms proportional to γ_5 have been eliminated through a chiral rotation; the Jacobian associated with such a transformation adds a term proportional to $W\tilde{W}$ to the Lagrangian and has no perturbative effects [8]. Within each irreducible representation the mass submatrix is proportional to the identity. If there are two multiplets ψ_1 and ψ_2 transforming according to the same irreducible representation we can replace them by arbitrary (unitary) linear combinations, and so we can choose M to be diagonal and real. It follows that this term is CP invariant. In contrast, terms of the form $\bar{\psi} \psi \Gamma \phi$ (where Γ denotes the Yukawa coupling matrix) are not, in general, CP invariant.

Finally, using the fact that the H_i are real and diagonal together with (9) we obtain

$$D_\mu \phi \rightarrow (D^\mu \phi)^* \tag{14}$$

which shows that the scalar kinetic Lagrangian $(D_\mu \phi)^\dagger D^\mu \phi$ is also an invariant. Just as for the fermion mass we can choose the scalar mass matrix \mathcal{M}^2 to be real and diagonal. It then follows that the term $\phi^\dagger \mathcal{M}^2 \phi$ will be invariant under CP.

These results show that, in the absence of spontaneous symmetry breaking, the only renormalizable terms that can violate CP in the Lagrangian are the fermion-scalar interactions (Yukawa couplings) and the scalar potential V .

d. Virtual CP-violating effects We now imagine that the Standard Model is the low-energy limit of a weakly-coupled natural gauge theory. In this case the corrections produced by virtual heavy physics effects to low-energy processes are determined by all gauge-invariant local operators involving the Standard Model fields. Leading effects are produced by operators with the largest coefficients. For the class of theories under consideration these are operators generated at tree-level [9].

We apply the previous result to this case as follows: we first isolate the effective operators that violate CP and whose coefficients are not suppressed by loop effects or constrained by existing data. These operators are of interest because, if present, they would provide the strongest non-standard CP-violating signals. We then describe the types of heavy physics that can generate these operators. At this point we can restrict ourselves to the case where the *heavy* physics is described by a renormalizable theory. Indeed, if the heavy scale is denoted by Λ , any operator of dimension higher than 4 present at this scale would be generated by interactions whose scale Λ' is larger than Λ , and the corresponding coefficient will be suppressed by an inverse power of Λ' ; at low energies the corresponding effects would be suppressed by a power of $\Lambda/\Lambda' \ll 1$ and can be ignored.

The specific form of these operators depends, of course, on the spectrum of light particles, we will assume that these include the usual Standard Model fermions and gauge bosons and, in addition, one scalar doublet. For this case the list of all tree-level-generated operators is known [9] and, using the above transformation rules, those which are not CP invariant can be isolated; they are³

²The term $W_{\mu\nu} \tilde{W}^{\mu\nu}$ is odd under CP but it is a total derivative and so has no effects to any finite order in perturbation theory.

³We denote the left-handed quark and lepton doublets by q and ℓ ; right-handed up and down quarks are denoted by u and d , right-handed charged leptons by e ; the scalar doublet is labeled ϕ . The generators of $SU(3)$ are denoted by λ^A , the ones for $SU(2)_L$ are the usual Pauli matrices τ^I , and $\varepsilon = i\tau^2$. D_μ denotes the covariant derivative and v the scalar doublet vacuum expectation value.

$$\begin{aligned}
I : & \left\{ \begin{array}{l} (\bar{\ell}e)(\bar{d}q) - \text{h.c.} \quad (\bar{q}u)\varepsilon(\bar{q}d) - \text{h.c.} \quad (\bar{q}\lambda^A u)\varepsilon(\bar{q}\lambda^A d) - \text{h.c.} \\ (\bar{\ell}e)\varepsilon(\bar{q}u) - \text{h.c.} \quad (\bar{\ell}u)\varepsilon(\bar{q}e) - \text{h.c.} \end{array} \right. \\
II : & \left\{ (|\phi|^2 - v^2)(\bar{\ell}e\phi - \text{h.c.}) \quad (|\phi|^2 - v^2)(\bar{q}u\tilde{\phi} - \text{h.c.}) \quad (|\phi|^2 - v^2)(\bar{q}d\phi - \text{h.c.}) \right. \\
III : & \left\{ (\bar{f}\gamma^\mu f)\partial_\mu(|\phi|^2 - v^2), \quad f = u, d, e, q, \ell \right. \\
IV : & \left\{ \begin{array}{l} \mathcal{O}_1 = (\phi^\dagger \tau^I \phi) D_\mu^{IJ} (\bar{q}\gamma^\mu \tau^J q) \\ \mathcal{O}_2 = (\phi^\dagger \tau^I \phi) D_\mu^{IJ} (\bar{\ell}\gamma^\mu \tau^J \ell) \\ \mathcal{O}_3 = (\phi^\dagger \varepsilon D_\mu \phi) (\bar{u}\gamma^\mu d) - \text{h.c} \end{array} \right. \quad (15)
\end{aligned}$$

each of the fields has, in principle a family index which we have suppressed.

Operators not included in the above list may arise from loop corrections, and so will have coefficients suppressed by a factors of $g^2/(4\pi)^2$, where g is some generic coupling constant. Their corresponding CP-violating effects are typically down by a factor $\sim g^2 v^2/(4\pi\Lambda)^2$ where $v \sim 246$ GeV denotes the vacuum expectation value of the Standard Model scalar doublet (we assume weakly interacting heavy physics). On the other hand, Standard Model CP violating effects are suppressed by a small factor $\sim 4 \times 10^{-5}$ proportional to a product of CKM matrix elements [10], so that the Standard Model effects will dominate for $\Lambda > 3$ TeV. Moreover it is possible for the underlying theory to have additional suppression factors not mandated by the gauge symmetry. So, even though loop generated operators may produce a wider variety of CP-violating signatures than those in (15), we expect the corresponding effects to be, at best, marginally inside the current sensitivity. For moderately heavy physics ($\Lambda > 3$ TeV) these small effects need to be disentangled from similar Standard Model contributions. We therefore conclude that the highest sensitivity to heavy CP-violating physics is obtained in reactions involving the operators in (15) to which we now turn.

The 4-fermion operators of group *I* in (15) contribute (at one loop) to the strong CP parameter θ and so their coefficients are strongly constrained (assuming no cancelations). Additional constraints can be obtained for fermions in the first generation from meson decays. Moreover, several of these operators (depending on the generation to which each of the fields belongs) will contribute radiatively to fermion masses, so that their coefficients will be suppressed by the (smallest) corresponding Yukawa coupling for natural heavy physics.

Operators involving scalars and fermions that violate chirality (group *II* in (15)) also contribute to the fermion masses and their coefficients are correspondingly suppressed in natural theories (except possibility for the ones contributing to the top-quark mass). Note also that operators in groups *II* and *III* in 15 can be probed only in processes involving real Higgs particles.

Thus the only unsuppressed operators that can be probed in current and near-future experiments and that do not involve scalars are those in group *IV*. These take the form (in unitary gauge)

$$\begin{aligned}
\mathcal{O}_1 & \rightarrow -\frac{igv^2}{\sqrt{2}} (\bar{u}_L W^+ d_L - \text{h.c.}) + \text{terms with scalars} \\
\mathcal{O}_2 & \rightarrow -\frac{igv^2}{\sqrt{2}} (\bar{\nu}_L W^+ e_L - \text{h.c.}) + \text{terms with scalars} \\
\mathcal{O}_3 & \rightarrow -\frac{igv^2}{\sqrt{8}} (\bar{u}_R W^+ d_R - \text{h.c.}) + \text{terms with scalars} \quad (16)
\end{aligned}$$

The Higgs-independent terms in $\mathcal{O}_{1,2}$ are indistinguishable from terms already present in the Standard Model. The Higgs-independent terms in the operators \mathcal{O}_1 generate a shift in the CKM matrix elements $V_{ij} \rightarrow V_{ij} + g(v/\Lambda)^2 v_{ij} + \mathcal{O}(1/\Lambda^4)$ which is not necessarily unitary.

For massless neutrinos a basis of lepton fields can be chosen so that the operators \mathcal{O}_2 do not mix families. The effects of the Higgs independent terms then correspond to a violation of universality in the couplings of the leptons to the W boson. Current data [11] require $\Lambda > 2.2$ TeV (at 1σ assuming the coefficient of the operator is $1/\Lambda^2$). For massive neutrinos the effects of these interactions correspond to a shift in the leptonic CKM matrix. Finally \mathcal{O}_3 corresponds to a right-handed quark current and contributes to the W mass and τ decay; existing data implies $\Lambda > 500$ GeV [11].

The possible tree-level graphs of the underlying theory that can generate $\mathcal{O}_{1,2,3}$ were determined in [9] and are shown in Fig. 1. In the following we call Φ a generic heavy scalar, F a heavy fermion, f a light (Standard Model) fermion and ϕ the Standard Model scalar doublet. Note that in view of the result proved above and in [1] we can immediately dismiss those graphs containing only gauge interactions as indicated in the figure. Moreover, knowing the structure of the graphs allows us to draw a number of general properties that the heavy fermions and scalars must satisfy in order to contribute to the operators of class *IV*.

Consider first the diagram containing a heavy scalar Φ , which must be an $SU(2)$ triplet and a $U(1)_Y$ singlet; one can also show that in order to maintain $v \ll \Lambda$ while having a Φ mass of order Λ , the triplet Φ must get a vacuum expectation value of (at most) order v^2/Λ . Given the previous transformation laws the $\phi\phi\Phi$ coupling is CP conserving,

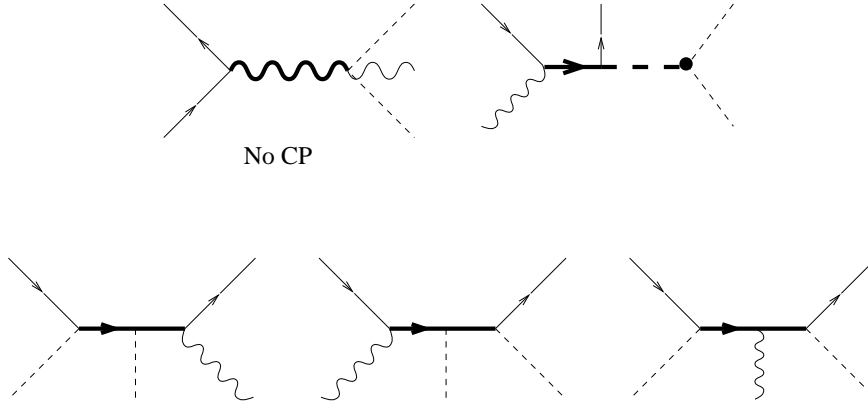


FIG. 1. Tree-level graphs that generate effective CP violation operators $\mathcal{O}_{1,2,3}$. Solid straight lines denote fermions, dashed lines denote scalars, wavy lines denote vector bosons. All thick lines represent heavy particles, the heavy dot denotes a super-renormalizable (SR) vertex $\sim \Lambda$.

hence the CP-violating properties of this graph can come only from the $Ff\phi$ couplings. For this coupling to exist the heavy fermions must be $SU(2)$ doublets, so that this graph cannot contribute to \mathcal{O}_3 .

Similar considerations apply to the remaining graphs that involve only heavy fermions. All involve a heavy fermion doublet (under $SU(2)$) and only the last graph can generate contributions to \mathcal{O}_3 . It follows that for natural, weakly-coupled heavy physics, observable CP violating interactions are generated by heavy fermions through their mixing with Standard Model fermions and/or through their Yukawa couplings with the Standard Model scalar doublet. In addition the Yukawa couplings of the form $Ff\phi$ are suppressed by a factor of the f mass (having assumed naturality) and so the most significant effects can be expected in processes involving the third generation of Standard Model fermions.

We conclude that for natural, weakly-coupled heavy physics the most significant CP-violating effects appear through violation of the unitarity relations in the CKM matrix for quarks (and, for massive neutrinos, for leptons also), in violations of universality in the coupling of the W to the fermions, or in the appearance of a right-handed charged current; these effects can be most noticeable when involving the top quark. We also note that stronger CP effects may be observed provided in reactions involving scalars. The above arguments also imply that existing data on right-handed currents can provide indirect bounds on the scale of heavy, CP-violating physics.

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